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Fracture toughness and absorbed energy measurements in impact tests on brittle materials

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Previous work on impact testing has shown that the energy/unit area (w) normally measured in notched impact tests is dependent on specimen geometry. A fracture mechanical analysis has now been developed to account for the observed dependence of w on notch size. A correction factor (ϕ) has been derived to accommodate notch effects and this allows for the calculation of the strain energy release-rate G directly from the measured fracture energies.

Tests on PMMA have shown that "corrected" results are independent of specimen geometry and the G_c for PMMA has been evaluated as 1.04×10^3 J m^{-2}. The experimental results show that there is an additional energy term which must be accounted for and this has been interpreted here as being due to kinetic energy losses in the specimens. A conservation of momentum analysis has allowed a realistic correction term to be calculated to include kinetic energy effects and the normalized experimental results show complete consistency between all the geometries used in the test series.

It is concluded that the analysis resolves many of the difficulties associated with notched impact testing and provides for the calculation of realistic fracture toughness parameters.

1. Introduction

In designing against the possibilities of premature fracture, one of the most important properties of a material is its ability to withstand impact. With many metals and most plastics, the susceptibility of the material to shock loading is often the most critical parameter considered in material selection. It therefore follows that it is essential to have a test/analysis combination which is able to give an accurate measure of impact fracture toughness so that different materials may be compared. At the present time the most favoured types of test are those involving the use of pendulum impact machines where the differences in initial and final potential energies of the pendulum are easily measured. The two most common types of test (Izod and Charpy) involve the bending of notched specimens, the primary differences in test method being in the mode of specimen support and point of pendulum contact on the specimen. Unfortunately, the "fracture energy" measured in both tests is not a material property since it varies with the proportions and size of test specimen; even for a given impact velocity, and tests made on specimens of different geometries often rate materials in different orders of merit. As an alternative it is sometimes advocated that the specific fracture energy, w/A (where w is the energy lost by the pendulum and A is the cross sectional area of the fractured ligament) is a reliable measure of material toughness [1, 2]. However, this parameter is strongly dependent on notch length and must therefore be excluded as a candidate toughness parameter.

The purpose of the present paper is to examine the nature of the geometrical and notch-length effects in the Charpy loading situation and to see whether some other simple correlation can be derived between w and the true fracture toughness.

The analytical relations which are derived in the following sections have been tested using PMMA as a model material since plastics are relatively easy to test in impact because of the low load levels required for fracture and also because there is a particularly serious impact problem in the practical applications of these materials.

2. The relationship between fracture toughness and absorbed energy

The strain energy, U, per unit thickness, absorbed in deflecting a cracked elastic test piece of thickness B is given by:

$$U = P\triangle/2B \tag{1}$$

where P is load and $\triangle$ the deflection of the point of application of the load.

If a crack of length a is extended by an amount da, the strain energy release rate, G, per unit thickness is:

$$G \equiv \mathrm{d}U/\mathrm{d}a = [P\mathrm{d}\triangle/\mathrm{d}a + (\triangle \mathrm{d}P/\mathrm{d}a)]/2B\ . \tag{2}$$

Defining compliance, C, as:

$$C = \triangle/P \tag{3}$$

and noting

$$\mathrm{d}C/\mathrm{d}a = (1/P)\,\mathrm{d}\triangle/\mathrm{d}a - (\triangle/P^2)\,\mathrm{d}P/\mathrm{d}a \tag{4}$$

then at constant load, P,

$$G = (P^2\mathrm{d}C/\mathrm{d}a)/2B\ . \tag{5}$$

Related expressions can be obtained for release rate at constant deflection, $\triangle$, or the rate expressed in terms of stiffness instead of compliance but this form is convenient here.

From the well-known Griffith-Irwin development of linear elastic fracture mechanics (for example, [3]):

$$G = K^2/E' \tag{6}$$

where E' is the reduced Young's modulus, E for plane stress, $E/1 - \nu^2$ for plane strain (ν is Poisson's ratio) and K is the stress-field intensity factor, characterizing the singularity of stress around a sharp crack.

At fracture, a critical value of energy release rate, G_c, or correspondingly K_c, is required, where G_c is the effective surface energy analogous to twice the surface energy in Griffith's original formulation, but including whatever dissipative plastic crack terms may arise local to the crack. K_c is commonly called the "fracture toughness" and designated K_{1c} the "plane strain fracture toughness" if a flat opening mode fracture under plane strain conditions is in question. In plane shear or out of plane "cross slip", fractures can also occur.

The fracture toughness is normally measured by observing the load, P, at which brittle fracture occurs in a suitable sharp-notch test piece, for example a three-point bend piece, for which the stress intensity factor K is known. In general:

$$K = Y\sigma\sqrt{a} \tag{7}$$

where Y is a geometrical factor introduced to account for finite width effects. For the infinite plate with centre crack $2a$ and remote uniform stress σ considered by Griffith, $Y = \sqrt{\pi}$. For the three-point bend piece of thickness B, width W, notch depth a, and span $S = 4W$:

$$Y = 1.93 - 3.07(a/W) + 14.53(a/W)^2 - 25.11(a/W)^3 + 28.80(a/W)^4\ . \tag{8}$$

Similar expressions with slightly-altered coefficients are given [4] for pure bending and three-point bend with $S = 8W$.

For three-point bend the nominal stress σ is given by simple bending theory as:

$$\sigma = 6PS/4BW^2\ . \tag{9}$$

Combining Equations 6 and 7 for the point of fracture gives:

$$G_c = (\mathrm{d}U/\mathrm{d}a)_f \tag{10}$$

$$= (Y^2\sigma^2\, a/E')_f \tag{11}$$

where the suffix f implies that the terms are evaluated at the instant of fracture.

Conventional use of the simple expression (mentioned in the Introduction) namely $G_c = w/A$ (where $A = B(W - a)$) instead of Equation 10 implies that the rate d/da at the onset of fracture equals the mean rate over the whole fracture path, i.e., that the crack movement "da" is in fact the whole ligament width ($W - a$) and also that:

$$w/B = U \tag{12}$$

(note that G and U are already on a unit thickness basis whereas w is not).

In general neither of these steps is valid, and although on occasions the numerical result may not be too far removed from a constant value of G_c, in other cases it may be in error by a factor of 5 or so.

The compliance, C, can be obtained by integrating Equation 5 if G is first expressed in terms of Y using Equations 6 and 7. Carrying this out for the three-point bend case using Equation 9 gives:

$$C = \frac{9S^2}{2BW^4E'}\int Y^2\, a\mathrm{d}a + C_0 \tag{13}$$

where C_0 is the compliance for zero crack length. From a conventional theory, for three-point bending:

$$C_0 = S^3/4EBW^3\ . \tag{14}$$

Thus if the only energy absorbed in a test, w, were in fact the elastic strain energy, UB, then since from Equations 1 and 3:

$$U = P^2C/2B \tag{15}$$

then by substituting for C from Equations 13 and 14, and expressing P in terms of σ from Equation 9 (and hence K from Equation 7) gives:

$$w = GB\left[\int Y^2 a\mathrm{d}a + (SW/18)\right] / Y^2a \tag{16}$$

$$= GBW\phi \tag{17}$$

where

$$\phi = \left[\int Y^2 x\,\mathrm{d}x + (S/18W)\right] / Y^2x \tag{18}$$

where $x = a/W$ the non-dimensional crack length.

The quantity ϕ is shown as a function of a/W in Fig. 1.

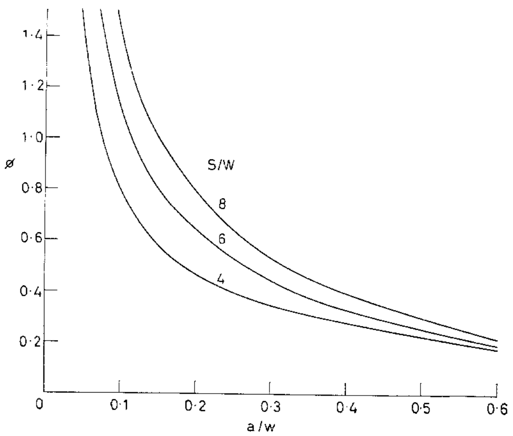

Figure 1 Correction factor (ϕ) versus non-dimensional crack length (a/w).

Since w is the energy which should be measured by the loss of potential energy of the pendulum in a conventional impact test, and at fracture, $G = G_c$ then:

$$G_c = w/BW\phi = w/A'\phi \tag{19}$$

(where A' is the gross cross-section of the test piece, not just the ligament area A) instead of just $G_c = w/A$ as in Equation 13. Clearly a plot of w against $BW\phi$ should be linear and of slope G_c.

To test the analysis, a programme was conceived using PMMA as a "model" material, the prime object of the tests being the assessment of the ϕ correction factor for a range of tests using different initial notch lengths in various specimen geometries.

3. Impact testing of PMMA

3.1. Test conditions

The tests were all conducted on a conventional Charpy-type pendulum impact machine which gave an impact velocity of 2.5 msec^{-1} and was supplied with a range of pendulums weighing from 0.07 to 2.2 kg, to suit plastics of various degrees of toughness. Energy losses from windage and friction in this machine were automatically allowed for on the pendulum scale. The striking edge of each pendulum had a double nose, the edges being 10 mm apart, so that the test specimens were all loaded in four-point bending. However, the central part of the span here is so small that it is doubtful whether a pure bending-stress system is ever fully developed so that three point bending is probably a closer approximation to the real situation.

Since one of the primary variables which was to be tested was the effects of specimen geometry, the tests were conducted on specimens machined from cast PMMA (I.C.I. "Perspex") sheet to the following dimensions (W:B:S):

(i) 6.4 × 6.4 × 45 mm (iv) 9.5 × 3.2 × 45 mm
(ii) 9.5 × 9.5 × 45 mm (v) 9.5 × 1.6 × 45 mm
(iii) 9.5 × 6.4 × 45 mm (vi) 3.2 × 9.5 × 45 mm.

The theory developed in the preceding section is viable for the particular case of sharp-notched specimens and accordingly each specimen was sharply notched by the slow, controlled insertion of a razor blade, the initial crack propagating ahead of the blade as it was forced into the material. A large number of specimens containing various initial crack lengths ($0.03 < a/W < 0.5$) were tested with each specimen geometry so as to assess the influence of notch length on the results. It was found that a crack length of 0.25 mm was the minimum which could be inserted to give a square uniform front across the section and this value was adopted as a standard lower-limit value.

All tests were conducted in an air-conditioned laboratory at 20° C and 50% relative humidity.

3.2. Test results

It was found that a number of test variables could affect the reproducibility of the data in this type of test. The quality and consistency of the initial pre-notching and the alignment of the specimen in the machine were observed to be critical in producing reproducible data since any variation in either could produce fractures which were not square across the section (Fig. 2) and absorbed-energy values in these cases were consistently

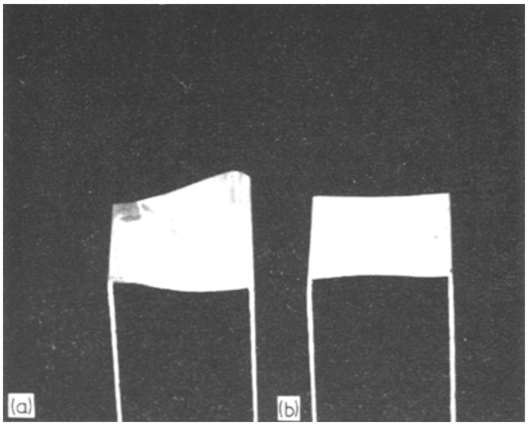

Figure 2 Types of fracture in PMMA (a) "warped" fracture, (b) flat fracture.

higher than for those tests where flat fractures occurred, and consequently the results of such tests were ignored.

As previously noted by many other workers, it was found that there was a strong dependence of the energy to fracture, w, on the size of the initial crack length. A typical graph of w/ligament area (A) versus notch depth ratio a/W is shown as Fig. 3. However, when the same results were plotted in the form w versus $BD\phi$ (Fig. 4), following Equation 15, the notch length dependency was seen to disappear and the results followed a predominantly linear pattern as expected. Contrary to expectation, however, the line did not pass through the origin of the graph since a least squares fit to the data showed that there was a positive intercept w' on the energy axis implying that there are other energy terms to be considered than just w and U. Nonetheless, it was pleasing to note that the slopes of the lines for the different specimen geometries were very consistent, thereby implying a constant value of G_c independent of both notch length and specimen size. The numerical values of the slopes and the w' energy intercepts for each geometry are summarized in Table I.

TABLE I

Specimen size (mm)	G_c ($J\ m^{-2} \times 10^{-3}$)	Intercept (w') (J)
$9.5 \times 9.5 \times 45$	1.05	0.046
$9.5 \times 6.4 \times 45$	1.01	0.025
$9.5 \times 3.2 \times 45$	1.03	0.009
$9.5 \times 1.6 \times 45$	0.97	0.005
$6.4 \times 6.4 \times 45$	1.03	0.014
$3.2 \times 9.5 \times 45$	0.99	0.005

4. Discussion of results

From the results of Fig. 4, it is clear that a linear relationship exists between w and $BD\phi$ (as predicted by Equation 15). The slopes of the graphs listed in Table I give values which should be G_c and it is to be noted that these numbers are consistent with data found elsewhere [5] so that the neglect of ϕ is the major unwarrantable step of principle in attempting to use w/A for the fracture toughness. It is seen however, that the

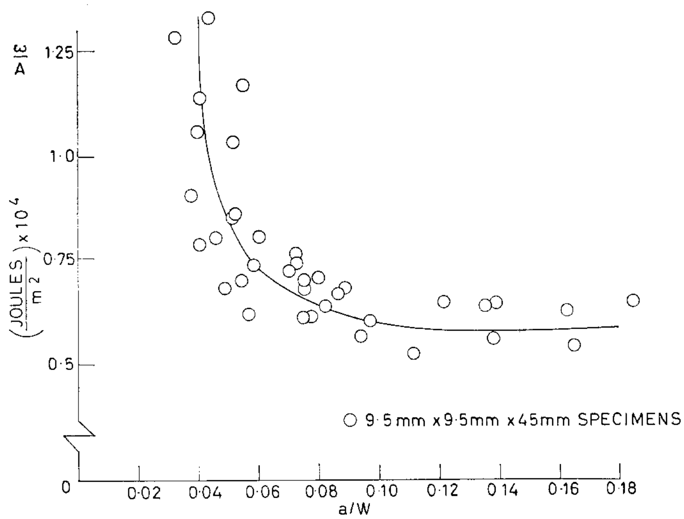

Figure 3 Specific "fracture energy" (w/a) versus non-dimensional crack length (a/w).

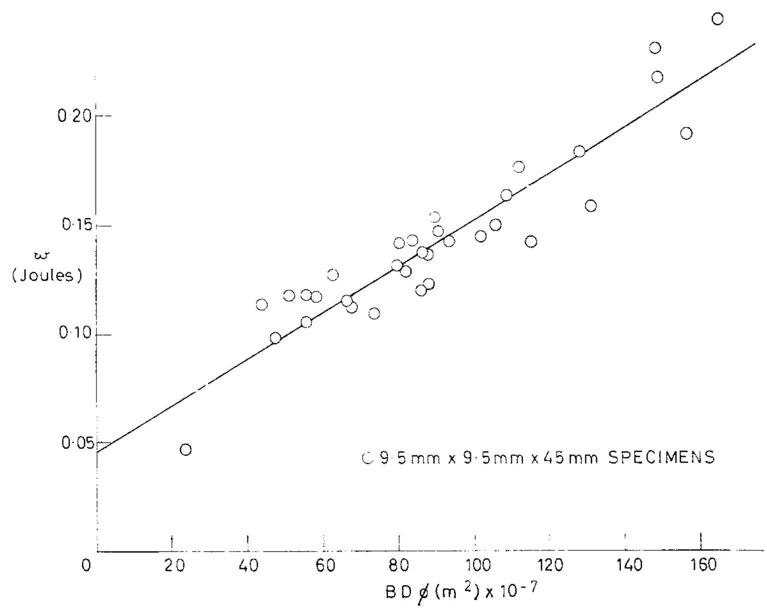

Figure 4 Measured energy (w) versus $BD\phi$.

use of $w/A'\phi$ from a single test will still not give a correct value of G_c because of the additional energy term which is included in the results.

The source of this apparent discrepancy lies in the assumption of Equation 12 that $w/B = U$ (for convenience in the following argument w is now taken on a unit thickness basis) since in an impact test of the nature described here there are numerous other energy "sinks" which require consideration. It is assumed here that only brittle fractures are being discussed and that such fractures occur rapidly from an elastic loading condition apart from plasticity very local to the crack tip. Clearly, if there is overall plasticity and a partly ductile fracture, plastic energy is absorbed in addition to the elastic strain energy, U. There are also possible friction losses between test piece and strikers or abutments, particularly if a ductile test piece is bent round the pendulum and forced through the abutments. A stable crack-growth for reasons of crack blunting, bifurcation or even rapid increase of G_c with strain-rate could also absorb energy subsequent to the initial fracture instability and such cases are excluded from the present argument and are thought not to be relevant to impact tests using brittle materials such as PMMA. There are possible small losses from friction between striker and test piece even in a brittle fracture, but these are thought to be negligible here since no indentation of the abutments is visible and the test piece does not "wrap round" the pendulum.

A complete study of the vibration of the specimen after impact and the corresponding interchange of potential, strain and kinetic energy is beyond the scope of this paper. This transient vibration has been studied [6] in relation to the force measured on an instrumented pendulum [7, 8]. The energy interchanges were not there considered, although the correction for inertia loading, [8], was found to be a function of a/W by means of the present compliance factor, ϕ, and the departure of G_c from w/A was noted [6]. In addition to the above sources of energy loss, the most obvious and overriding additional term to be considered is the kinetic energy imparted to the test specimen during impact. All the kinetic energy transferred to the test piece first enters as strain energy, since momentum is given to the outer extremities by shear waves passing outward along the beam. It is suggested, however, that it is incorrect to infer that all the kinetic energy is thus a "consequence" of strain energy and should therefore be neglected. The time distribution of U and k must be considered, albeit quite approximately.

At the instant of fracture, some time t after the initial impact (perhaps a few milliseconds) some kinetic energy (k_t) will already exist, having passed into the test piece via strain energy. Thus, whilst over the completed process, all the kinetic energy is *in consequence* of strain energy, the strain energy available to cause fracture at time t is:

$$U = w - k_t \,. \tag{20}$$

This perhaps begs the issue of whether some at least of the kinetic energy in the test piece is not also available to cause fracture. A more detailed study would be required to resolve this point. The Griffith concept is, however, essentially a static one and a quasi-static solution is being sought here. To employ Equation 20, an estimate of k_t is required. It may be noted however that although this argument suggests that the magnitude of w is incorrect for use in $\mathrm{d}U/\mathrm{d}a$, giving rise to the intercept on the graphs such as Fig. 4, the slope implied by Equation 19 will still give a good value of G_c if k_t is sensibly constant for the tests with various notch depths.

From conventional mechanics calculations for impact of solid bodies, the ejection velocity of the test piece may well be much greater than the end velocity of the pendulum for the predominantly elastic behaviour associated with brittle fractures. Those calculations and some of the tests described below both show that the magnitude of k_t is by no means negligible, thus giving general support to the arguments advanced here.

5. Assessment of kinetic energy loss (k_t)

Although the absolute magnitude of the kinetic-energy loss-term, k_t, may be open to question, it is suggested that a good estimate may be made by measuring the kinetic energy loss in ejecting an unbroken specimen from the test machine, since k_t is notionally assessed immediately following impact and before total fracture has occurred.

The measured scale values of the energy loss on an unbroken specimen will, of course, depend on the relative sizes of the specimen and pendulum and a simple correction must be made to account for these quantities.

From classical mechanics, a mass M (the pendulum) striking with velocity V, a mass m (the test piece, notionally unsupported) at rest, will impart to it a velocity v where:

$$v = V(m/m + M)(1 + e) \tag{21}$$

(e is the coefficient of restitution).

The kinetic energy of the test piece (k_e) is then:

$$k_e = V^2(1 + e)^2\, mM^2/2(m + M)^2 \tag{22}$$

To confirm this relationship, tests were made on all the sizes of test pieces used in the main series, the specimens being ejected from the machine using the full range of pendulums. The measure of k_e was taken as the loss of potential energy recorded by the swing of the pendulums. In all the tests, the specimens were ejected between 0.7 and 1.5 m (depending on the specimen and pendulum size) in front of the machine, rising approximately 15 cm in the air in mid-path. A graph of k_e against $mM^2/(m + M)^2$ gave one straight line, Fig. 5, whence using Equation 22 for the known initial velocity of 2.5 m sec^{-1} gives $e = 0.776$. The line for perfect collision, $e = 1$, is also shown.

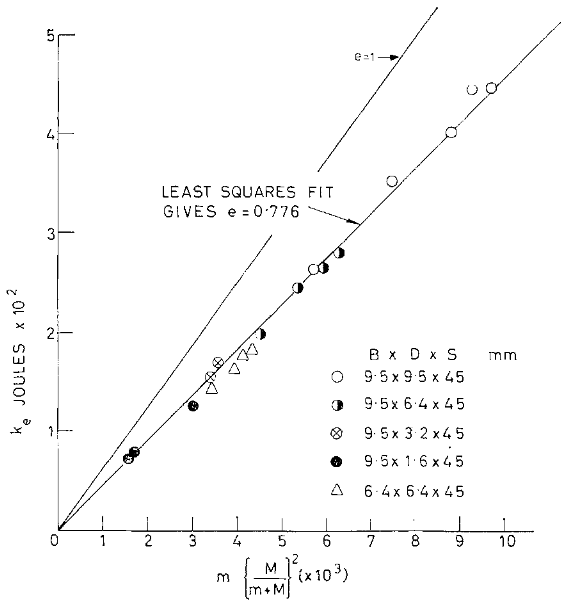

Figure 5 Kinetic energy of unnotched specimens versus $mM^2/(m + M)^2$.

To check on the magnitude and validity of using the term k_e as a measure of k_t, the original results for the fractured specimens were recomputed by using a modified version of Equation 19, namely

$$G_c = w^*/BW\phi \tag{23}$$

where

$$w^* = w - \tfrac{1}{2}mv'^2$$

$$= w - \tfrac{1}{2}(1 + e)^2 \,.\, m\left(\frac{M}{m + M}\right)^2 V^2 \,. \tag{24}$$

A composite graph of w^* versus $BD\phi$ for all the results on the different specimen geometries is shown in Fig. 6. It can be seen that the

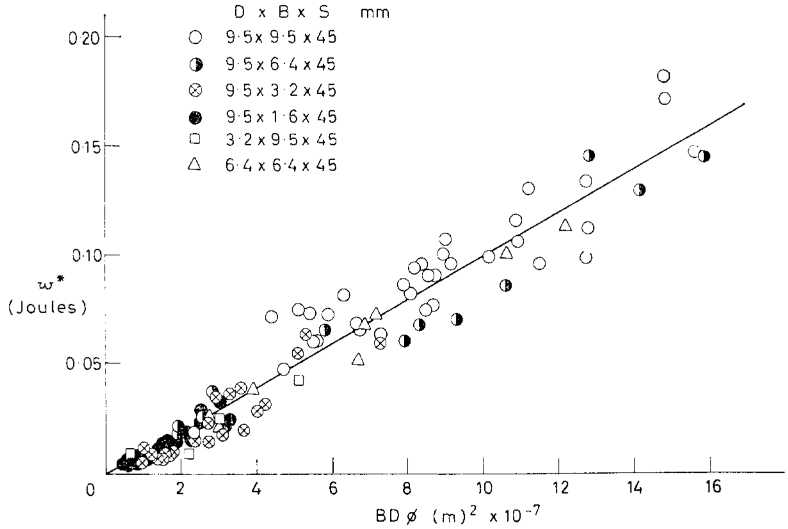

Figure 6 Corrected energy w^* versus $BD\phi$: all specimens.

correction term is of the right order of magnitude in that the best straight line, giving G_c, passes through the origin of the graph. The slope of this line gives $G_c = 1.04 \times 10^3$ J m^{-2} which is to be compared with the uncorrected results of Table I, showing that the correction term has essentially left G_c unchanged. Further, the results on individual specimens can now possibly be used for an assessment of G_c since there is no additional energy term unaccounted for.

It is therefore concluded that, as a good guide, kinetic energy terms comparable to those for perfect impact are generated without recourse to the bending strain energy caused by the abutment reaction, so that k_e represents a reasonable estimate of k_t. It is to be noted that the present analysis discounts rotational energy terms and it may be that if these are included then the scatter shown in Fig. 6 may be considerably reduced.

The practical objective of obtaining G_c from a single measurement of energy would therefore appear to be feasible providing the calibration term ϕ and the kinetic energy losses are accounted for. It is suggested that if several tests are to be carried out to allow for scatter on results, it is better to conduct these tests using specimens with various notch depths rather than simply repeating tests for one particular notch depth. Then, if brittle fractures occur, plots such as Figs. 4 and 6 would enable the determination of G_c. For plastics, it may well be found that G_c values will vary depending on the effective strain-rate applied to the specimen since the value of $G_c = 1.04 \times 10^3$ J m^{-2} for PMMA is indicative of an initial crack speed of 300 m sec^{-1}, which in tensile tests would be expected to occur for cross-head rates $\risingdotseq$ 500 m min^{-1}.

6. Conclusions

The used energy per unit area, w/A, is not justified as a measured toughness since this term is geometry dependent. The geometry dependence, ϕ, can be evaluated (Fig. 1 here) so that an expression $G_c = w/BW\phi$ is obtained. An additional factor entering into impact tests is that some energy, k_t, is converted into kinetic energy before fracture occurs so that a term $(w - k_t)$ is required, k_t being assessed by simple tests on unnotched specimens. If tests on pieces of various notch depths are made, G_c can then be obtained directly from the slope of a graph of $(w - k_t)$ against $BW\phi$. These remarks apply in principle only to classic brittle fractures since any significant ductility or other form of energy absorption such as crack blunting subsequent to the initial fracture will increase the energy absorbed over and above the value of w relevant to the elastic analysis. Values of toughness obtained by this method are directly comparable to values of K obtained directly from the fracture load.

Whilst this work was being completed, the authors learned of very similar work being conducted by Dr H. Brown, Physics Department, University of Leeds.

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